HOMEWORK #6:
5.10, 6.2, 6.3, 6.4, 6.5

$\boxed{5.10}$ $q, v$

- density: $N_i + N_e$
- K temp: $T_i \approx T_e = T$
- $P_{rad} \propto q^2 |dv/dt|^2$

Can't neglect ions, eqn (5.82)

a) $N_D = N_H \Rightarrow N_D = N = N_H$

$N_e = \frac{3}{2} N_i$

$P_{cyc} \propto N\left(\frac{q^4 B^2}{m^2}\right)(KT)$

Deuterium:

$P_{cyc} \propto \frac{N}{2}\left(\frac{(1)\ B^2}{(3.3446\times10^{-27}kg)^2}\right) KT$

Helium

$P_{cyc} \propto \frac{N}{2}\left(\frac{2^4\ B^2}{(5.0084\times10^{-27}kg)^2}\right) KT$

$P_{cyc,ions} = \frac{N}{2} B^2 KT\left(\frac{1}{1.187\times10^{-53}} + \frac{16}{2.5084\times10^{-53}}\right)$

$= 3.610\times10^{53}\ B^2 \cdot KT \cdot N$

electrons

$P_{cyc} \propto \frac{3}{2} N\left(\frac{B^2}{(9.1095\times10^{-31}kg)^2}\right) KT$

$P_{cyc} \propto \frac{3}{2} NB^2 KT\ (1.20507\times10^{60}) = 1.8076\times10^{60}\ NB^2 KT$

$\frac{P_e}{P_{ions}} = \left(\frac{\cancel{NKTB^2}}{\cancel{NKTB^2}}\right)\left(\frac{1.8076\times10^{60}}{3.610\times10^{53}}\right) = 5.0072\times10^{6}$

$\therefore$ $\boxed{P_{cyc,ions} \text{ is } 5.0\times10^{6} \text{ times smaller}}$

b) $P_{cyc} = (6.3\times10^{-20}\frac{J}{eV \cdot T^2 s})(6\,Tesla)^2 (80000\,eV)(2\times10^{20}\frac{1}{m^3})(3)$

$\boxed{P_{cyc} = 1.089\times10^{8}\ \frac{W}{m^3}}$

$\sigma N_D q$

$P_{fu} = \langle \sigma v \rangle N_D N_H Q$

$= (1.27\times10^{-22})\frac{m^3}{s}(2\times10^{20}\frac{ions}{m^3})^2 (18.3\ MeV)\times\left(\frac{1\times10^6 \cdot 1.6022\times10^{-19}J}{1\ MeV}\right)$

$\boxed{P_{fu} = 1.49\times10^{7}\ \frac{W}{m^3}}$

$P_{cyc}$ is 7.3 times greater than $P_{fu}$. This means that radiation reflecting on walls can not be neglected because it is a significant amount in break even

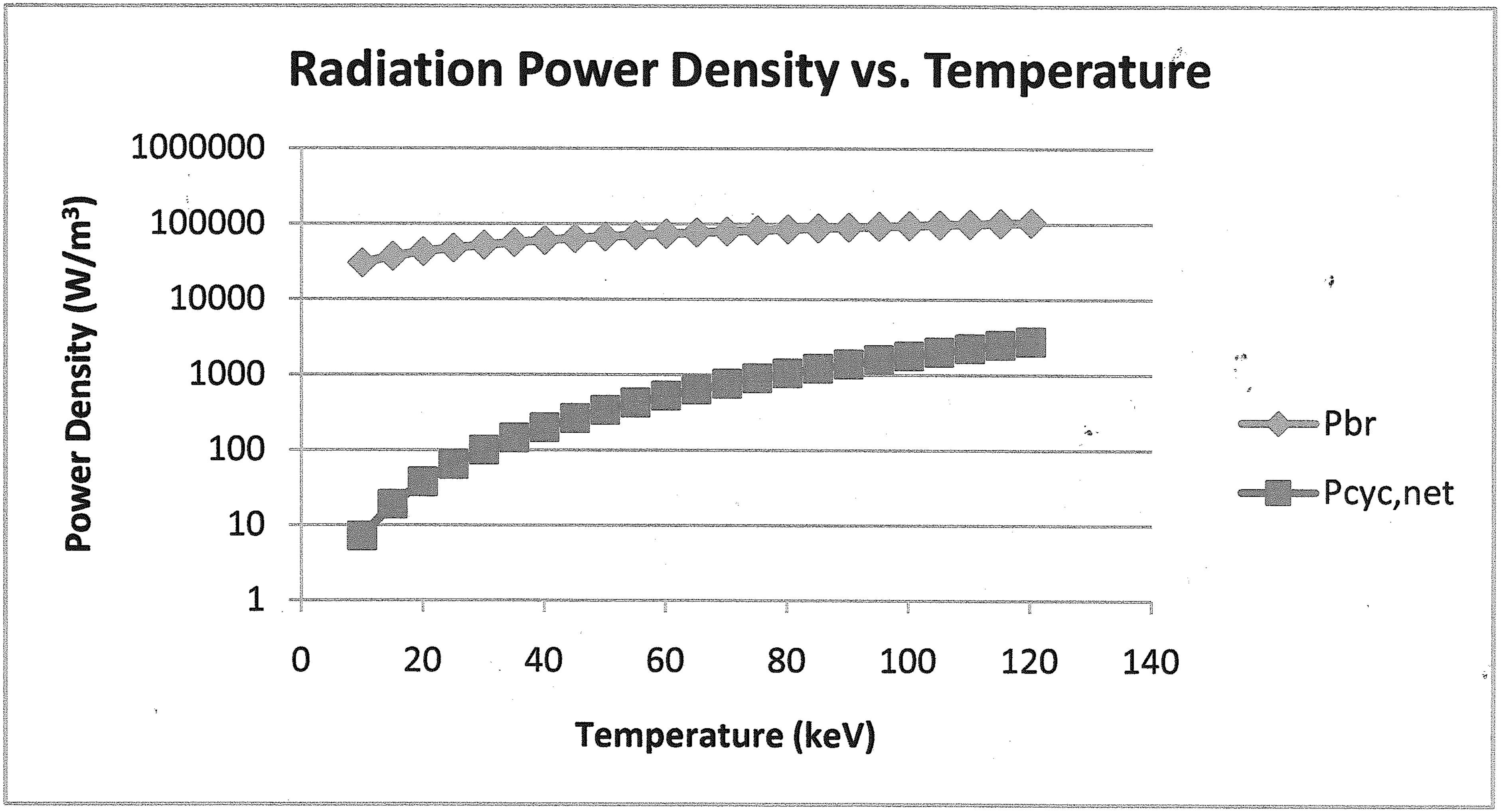
c) $\Psi(T) \approx 10^{-4.2} \left[ T(\mathrm{keV}) / 1\,\mathrm{keV} \right]^{1.4}$ $\quad T \rightarrow 10-120\,\mathrm{keV}$

$$P_{cyc} = A_{cyc} N_c B^2 k T_e \psi$$

$$P_{cyc}^{net} = 453.6\, kT \left[ 10^{-4.2} \left[ \frac{kT}{1\,\mathrm{keV}} \right]^{1.4} \right]$$

$$P_{br} = A_{br} N_d N_e Z^2 \sqrt{kT} + A_{br} N_n N_e Z^2 \sqrt{kT}$$

$$P_{br} = 9.6 \times 10^{3} \sqrt{kT}$$


Radiation Power Density vs. Temperature
Power Density (W/m³)
1000000
100000
10000
1000
100
10
1
0
20
40
60
80
100
120
140
Temperature (keV)
Pbr
Pcyc,net

6.2 Transport equation (6.42)

$$\frac{\partial N}{\partial t}+\vec{V}\cdot\nabla_r N+\frac{q}{M}(\vec{E}+\vec{V}\times\vec{B})\cdot\nabla_V N=\left(\frac{\partial N}{\partial t}\right)_c+\left(\frac{\partial N}{\partial t}\right)_s$$

a) $q=0$, equation becomes

$$\frac{\partial N}{\partial t}+\vec{V}\cdot\nabla_r N=\left(\frac{\partial N}{\partial t}\right)_c+\left(\frac{\partial N}{\partial t}\right)_s$$

b) For monoenergetic particles, $E=$ constant.

$\Rightarrow$ $V=$ constant, i.e., $V_x^2+V_y^2+V_z^2=$ constant.

The transport equation becomes

$$\begin{cases}\frac{\partial N}{\partial t}+\vec{V}\cdot\nabla_r N+\frac{q}{M}(\vec{E}+\vec{V}\times\vec{B})\cdot\nabla_V N=\left(\frac{\partial N}{\partial t}\right)_c+\left(\frac{\partial N}{\partial t}\right)_s\\ V_x^2+V_y^2+V_z^2=\text{constant}=\frac{2E}{m}.\end{cases}$$

c) only the z-direction is relevant, implying $\frac{\partial N}{\partial x}=\frac{\partial N}{\partial y}=0$, $\frac{\partial N}{\partial V_x}=\frac{\partial N}{\partial V_y}=0$

$$\vec{V}\cdot\nabla_r N=V_x\cdot\underbrace{\cancel{\frac{\partial N}{\partial x}}}_{0}+V_y\cdot\underbrace{\cancel{\frac{\partial N}{\partial y}}}_{0}+V_z\cdot\frac{\partial N}{\partial z}=V_z\cdot\frac{\partial N}{\partial z}.$$

$$\vec{V}\times\vec{B}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\V_x&V_y&V_z\\B_x&B_y&B_z\end{vmatrix}=(V_yB_z-B_yV_z)\hat{i}+(V_zB_x-V_xB_z)\hat{j}+(V_xB_y-V_yB_x)\hat{k}$$

So, $(\vec{E}+\vec{V}\times\vec{B})\cdot\nabla_V N=E_z\cdot\frac{\partial N}{\partial V_z}+(V_xB_y-V_yB_x)\cdot\frac{\partial N}{\partial V_z}$.

$\Rightarrow$ Transport equation becomes

$$\frac{\partial N}{\partial t}+V_z\frac{\partial N}{\partial z}+\frac{q}{m}\left[E_z\frac{\partial N}{\partial V_z}+(V_xB_y-V_yB_x)\cdot\frac{\partial N}{\partial V_z}\right]=\left(\frac{\partial N}{\partial t}\right)_c+\left(\frac{\partial N}{\partial t}\right)_s$$

d) The equation is just the same as (6.42), since the source term and collision terms need to be considered.

6.3. Plasma diffusion follows

$$\frac{\partial}{\partial t}N - D\nabla_r^2 N = 0$$

For the one-dimension case, $\frac{\partial}{\partial t}N - D\frac{d^2N}{dx^2} = 0$.

Let $N(t,x) = T(t)\cdot R(x)$. — seperation of variables.

$$\therefore \frac{dT(t)}{dt}\cdot R(x) - D\cdot\frac{d^2R(x)}{dx^2}\cdot T(t) = 0$$

$$\frac{1}{T}\frac{dT}{dt} = D\cdot\frac{1}{R}\frac{d^2R(x)}{dx^2}$$

Let each side equals to $-\lambda^2$ (it equals to a negative value since density decays w/ time).

$$\frac{1}{T}\frac{dT}{dt} = -\lambda^2 \Rightarrow T = C_1 e^{-\lambda^2 t}$$

$$D\cdot\frac{1}{R}\frac{d^2R(x)}{dx^2} = -\lambda^2 \Rightarrow R = C_2\sin\left(\frac{\lambda}{\sqrt{D}}x + C_3\right)$$

Boundary conditions. $N(x=0) = N(x=x_0) = 0$.

$$\Rightarrow R(x=0) = R(x=x_0) = 0.$$

$$\Rightarrow \begin{cases} C_3 = 0 \\ \frac{\lambda}{\sqrt{D}}x_0 = \pi, \cancel{2\pi}, \cancel{3\pi} \end{cases}$$

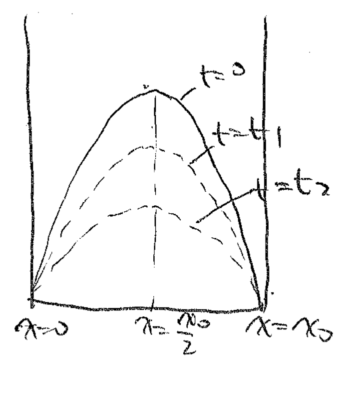
$$\Rightarrow \lambda = \frac{\sqrt{D}\,\pi}{x_0}$$

$$N(t,x) = C\cdot\sin\left(\frac{\pi x}{x_0}\right)\cdot e^{-\frac{D\pi^2}{x_0^2}t}$$

Initial condition. $N(t=0, x=\frac{x_0}{2}) = N_0$.

$$\Rightarrow C = N_0.$$

$$N(t,x) = N_0\sin\left(\frac{\pi x}{x_0}\right)e^{-\frac{D\pi^2}{x_0^2}t}$$



6.4. With the source term, (6.13) becomes

$$\frac{\partial}{\partial t}N(\vec{r},t) - D\nabla^2 N(\vec{r},t) = \underbrace{S}_{\text{constant}}$$

For steady state,

$$-D\nabla^2 N(\vec{r},t) = S$$

$$-D\cdot\frac{1}{r}\frac{d}{dr}\left(r\frac{dN}{dr}\right) = S$$

$$\Rightarrow \frac{d}{dr}\left(r\frac{dN}{dr}\right) = -\frac{S}{D}r.$$

$$\Rightarrow r\frac{dN}{dr} = -\frac{S}{2D}r^2 + C_1$$

$$\Rightarrow \frac{dN}{dr} = -\frac{S}{2D}r + \frac{C_1}{r}$$

$$\Rightarrow N = -\frac{S}{4D}r^2 + C_1\ln r + C_2$$

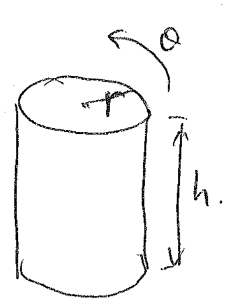
BC①. $N(r=0) \neq$ infinite $\Rightarrow C_1 = 0$.

BC② $N(r=0) = N(0) \Rightarrow C_2 = N(0)$

$$\underline{N = -\frac{S}{4D}r^2 + N(0)}$$

↳ We are not able to get (6.54) though.

6.5. You can either use the above relationship or (6.54).
Here I use (6.54).



$$\text{Average density} = \frac{\int N\,dV}{\int dV} = \frac{\int_0^a N\cdot r\,dr\cancel{\int_0^{2\pi}d\theta\int_0^H dh}}{\int_0^a r\,dr\cancel{\int_0^{2\pi}d\theta\int_0^H dh}}$$

$$= \frac{\int_0^a N(0)\sqrt{1-r^2/a^2}\cdot r\,dr}{\frac{a^2}{2}}$$

$$= \frac{\left(1-\frac{r^2}{a^2}\right)^{\frac{3}{2}}\cdot\left(-\frac{1}{3}a^2\right)N(0)\Big|_0^a}{a^2/2}$$

$$= \frac{2}{3}N(0)$$