2.2

$$u(R_0) = \frac{1}{4\pi\varepsilon_0} \frac{q_a q_b}{(R_a + R_b)}$$

$$R_a + R_b = R_0 = R_p(A_a^{1/3} + A_b^{1/3})$$

constants

$\varepsilon_0 = 8.854 \times 10^{-12} \frac{s^4 A^2}{m^3 kg}$ $\qquad R_p = 1.5 \times 10^{-15}$ m

d

$q = 1.6022 \times 10^{-19} C = 1 q_e$

$A_d = 2$

t

$q = 1 q_e$

$A_t = 3$

$$U_{dt} = \frac{1}{4\pi\varepsilon_0 R_0} = \boxed{0.355 \text{ MeV} = U_{dt}}$$

h

$q = 2$

$A_h = 3$

$$U_{dh} = \frac{2}{4\pi\varepsilon_0 R_0} = \boxed{0.711 \text{ MeV} = U_{dh}}$$

p

$q = 1$

$A_p = 1$

11B

$q = 5$

$A_{11B} = 11$

$$U_{p11B} = \frac{5}{4\pi\varepsilon_0 R_0} = \boxed{1.49 \text{ MeV}}$$

2.4

a)

$kT = 9\text{ keV}$

average energy $= \frac{3}{2} kT = \boxed{13.5\text{ keV}}$

b) average particle speed $= \sqrt{\frac{8}{m\pi}} \sqrt{kT}$

$\sqrt{kT} = \sqrt{9\text{keV}} = \sqrt{1.442 \times 10^{-15}\text{J}}$

$m = 5.0085 \times 10^{-27}\text{kg}$

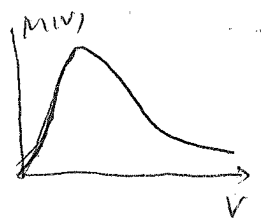
$\boxed{\bar{v} = 856.2 \frac{km}{s}}$

c) average energy $= \frac{1}{2} mv^2 = 1.8358 \times 10^{-15}\text{J} = \boxed{11.46\text{ keV}}$

The energy in c is lower than that of part a.
This is because in c only translational kinetic energy is taken into consideration, Part a takes into account rotations and vibrations.

The difference is because of two different distributions for v and E.

Fig 2.2



also. $\overline{v^2} \neq \bar{v}^2$

$$\int_0^\infty M(v)\,dv = \int_0^\infty M(E)\,dE$$

$$M(v)\,dv = M(E)\,dE \quad \left(M(E) = M(v)\frac{dv}{dE}\right)$$

$$\left[\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{m}{kT}\right)^{3/2} v^2\, e^{\left(\frac{-\frac{1}{2}mv^2}{kT}\right)}\right] dv$$

$$E = \frac{1}{2}mv^2 \qquad \frac{dE}{dv} = mv \qquad dv = \frac{dE}{mv}$$

$$\left[\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{m}{kT}\right)^{3/2} v^2\, e^{\left(\frac{-E}{kT}\right)}\right]\frac{dE}{mv} = \left[\left(\frac{2}{\pi}\right)^{1/2} m^{1/2}\left(\frac{1}{kT}\right)^{3/2} v\, e^{\left(\frac{-E}{kT}\right)}\right] dE$$

$$\times \quad \sqrt{\frac{2E}{m}} = v$$

$$\left[\left(\frac{2}{\pi}\right)^{1/2} m^{1/2}\left(\frac{1}{kT}\right)^{3/2}\left(\frac{2E}{m}\right)^{1/2} e^{\left(\frac{-E}{kT}\right)}\right] dE$$

$$\boxed{\frac{2}{\sqrt{\pi}}\left(\frac{1}{kT}\right)^{3/2} E^{1/2}\, e^{\left(\frac{-E}{kT}\right)}\, dE}$$

$$\boxed{M(E) = \frac{2}{\sqrt{\pi}}\left(\frac{1}{kT}\right)^{3/2} E^{1/2}\, e^{\left(\frac{-E}{kT}\right)}}$$

2.6

$$\int_{-\infty}^{\infty} M(\bar{v})\,d\bar{v} = \int_{0}^{\infty} M(\bar{v})v^2\,dv\,\sin\theta\,d\theta\,d\phi$$

$$\int_0^{\infty} M(\bar{v})v^2\,dv\;\underbrace{\int_0^{\pi}\sin\theta\,d\theta}_{2}\;\underbrace{\int_0^{2\pi}d\phi}_{2\pi}$$

$$4\pi\int_0^{\infty} M(\bar{v})\,v^2\,dv = \int_0^{\infty} 2^2\pi\left(\frac{m}{2\pi kT}\right)^{3/2} e^{\left(\frac{-\frac{1}{2}mv^2}{kT}\right)}v^2\,dv$$

$$\int_0^{\infty}(2^{1/2})\frac{1}{\sqrt{\pi}}\left(\frac{m}{kT}\right)^{3/2} e^{\left(\frac{-E}{kT}\right)}v^2\,dv$$

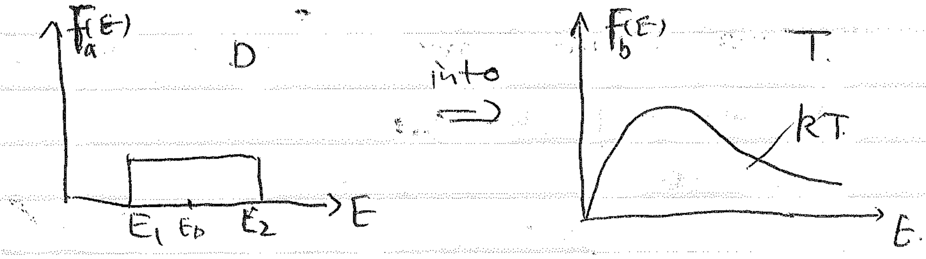
$$= \int_0^{\infty}\frac{\sqrt{2}}{\sqrt{\pi}}\left(\frac{m}{kT}\right)^{3/2} e^{\left(\frac{-E}{kT}\right)}\,v^{\cancel{2}}\,\frac{dE}{m\cancel{v}}$$

$$= \int_0^{\infty}\sqrt{\frac{2}{\pi}}\left(\frac{m}{kT}\right)^{3/2}\left(\frac{2E}{m}\right)^{1/2}\frac{1}{m}\;e^{\left(\frac{-E}{kT}\right)}\,dE$$

$$= \int_0^{\infty}\boxed{\frac{2}{\sqrt{\pi}}\left(\frac{1}{kT}\right)^{3/2}E^{1/2}\;e^{\left(\frac{-E}{kT}\right)}}\,dE$$

$$= M(E)$$

5) A beam of D injected into T plasma:



$$\langle \sigma v \rangle_{ab} = \int_{\vec{V_a}} \int_{\vec{V_b}} \sigma_{ab}(|\vec{V_a}-\vec{V_b}|) \, |\vec{V_a}-\vec{V_b}| \, F_a(\vec{V_a}) F_b(\vec{V_b}) \, d^3\vec{V_a} \, d^3\vec{V_b}$$

$\Rightarrow$ For the beam, assuming it's following x-direction
then $\vec{V_a} = V_{ax} \hat{e_x}$, and we only need to consider V_{ax}..

$$F(V_{ax}) = F_a(E) \frac{dE}{dV_{ax}} = F_a(E) \cdot m_D V_{ax}$$

$$F(E) = \frac{1}{E_2 - E_1} \quad (E_1 \le E \le E_2) \quad \left(\text{get it from } \int_0^\infty F(E)\, dE = 1\right)$$

$$\Rightarrow F(V_{ax}) = \frac{m_D V_{ax}}{E_2 - E_1} \quad \left(\sqrt{\frac{2E_1}{m_D}} \le V_{ax} \le \sqrt{\frac{2E_2}{m_D}}\right)$$

$\Rightarrow$ For the T plasma. $\vec{V_b} = V_{bx}\hat{e_x} + V_{by}\hat{e_y} + V_{bz}\hat{e_z}$

$$F_b(\vec{V_b}) = \text{Maxwell-Boltzmann distribution}$$

$$= \left(\frac{m_T}{2\pi kT}\right)^{3/2} \exp\left(\frac{-\frac{1}{2} m_T V^2}{kT}\right)$$

$$= \left(\frac{m_T}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m_T (V_{bx}^2 + V_{by}^2 + V_{bz}^2)}{2kT}\right)$$

$$\Rightarrow |\vec{V_a} - \vec{V_b}| = \sqrt{(V_{ax} - V_{bx})^2 + V_{by}^2 + V_{bz}^2}$$

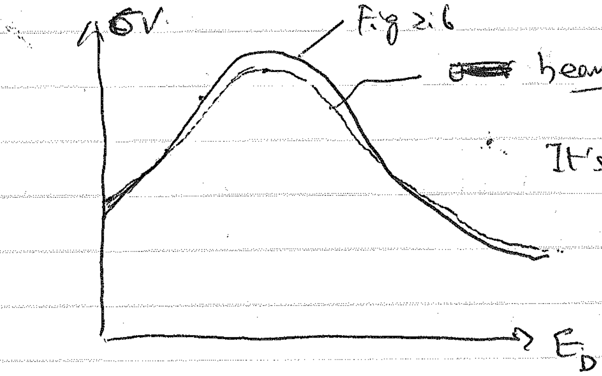
So, $$\langle \sigma v \rangle_{ab} = \int_{\sqrt{\frac{2E_1}{2m_D}}}^{\sqrt{\frac{2E_2}{m_D}}} dV_{ax} \int_0^\infty \int_0^\infty \int_0^\infty dV_{bx}\, dV_{by}\, dV_{bz} \cdot$$

$$\sigma_{ab}(|\vec{V_a}-\vec{V_b}|) \cdot |\vec{V_a}-\vec{V_b}| \, \frac{m_D V_{ax}}{E_2 - E_1} \cdot \left(\frac{m_T}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m_T (V_{bx}^2 + V_{by}^2 + V_{bz}^2)}{2kT}\right)$$

as shown above

How does it compare to Fig 2.6?

Remember Fig 2.6 is for a monoenergetic D beam injected into T plasma ...



beam $E_1 < E < E_2$

$E_D = \frac{E_1+E_2}{2}$

It's a convolution of $F_a(E)$ over $\langle \sigma v \rangle_{ab}$ in Fig 2.6, or an averge between. A range of $E_2 - E_1$ for every point.

So, the peak point is lowered, and the whole curve is smoothened.

The optimal point for E_D will depend on two ~~things~~ things.

1. high $\langle \sigma v \rangle$
2. As low E_D as possible.

Based on Fig 2.6, 60 keV might be the optimal point.