Homework #3

3.2) Eqn 3.15

$$\sigma_s(\theta_c) = \frac{1}{4}\left(\frac{q_\alpha q_c}{4\pi \varepsilon_0 m_r v_r^2}\right)^2 \frac{1}{\sin^4(\theta_c/2)}$$

$$m_r = \frac{(m_\alpha m_c)}{(m_\alpha + m_c)} = \frac{(3.3446\times10^{-27}\,kg)(5.0085\times10^{-27}\,kg)}{(3.3446\times10^{-27}\,kg + 5.0085\times10^{-27}\,kg)}$$

$$m_r = 2.005\times10^{-27}\ kg$$

V_α: $E = \frac{1}{2}mv^2$

$$5\,keV = \tfrac{1}{2}(3.3446\times10^{-27}\,kg)\,v^2$$

$$8.011\times10^{-16}\,J = 1.6723\times10^{-27}\,kg\cdot v^2$$

$$V_\alpha = 6.922\times10^5\ m/s$$

V_c: $E = \frac{1}{2}mv^2$

$$5\,keV = \tfrac{1}{2}(5.0085\times10^{-27}\,kg)\,v^2$$

$$8.011\times10^{-16}\,J = 2.50425\times10^{-27}\,kg\cdot v^2$$

$$V_c = 5.656\times10^5\ m/s$$

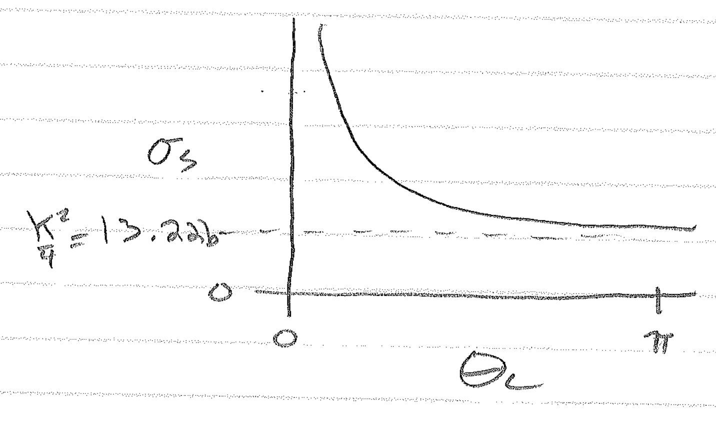
$$V_r = |6.922\times10^5\ m/s + 5.656\times10^5\ m/s|$$

$$V_r = 1.2578\times10^6\ m/s$$

$$\sigma_s(\theta_c) = \frac{1}{4}\left(\frac{(1.6022\times10^{-19})^2}{4\pi(8.8542\times10^{-12})(1.2578\times10^6)^2(2.005\times10^{-27})}\right)^2 \frac{1}{\sin^4(\theta_c/2)}$$

$$\sigma_s(\theta_c) = (1.3226\times10^{-27})\,(1/\sin^4(\theta_c/2))$$

$$\sigma_s(\theta_c) = 13.226\,(1/\sin^4(\theta_c/2))\ \text{barns}$$



$\boxed{3.4}$ $P_{br} = A_{br} N_i N_e Z^2 \sqrt{kT}$

Case 1:

$N_D = N_T = \frac{1}{2} N_e$

$$P_{br} = A_{br}\sqrt{kT}\,(N_D N_e 1^2 + N_T N_e 1^2)$$
$$= A_{br}\sqrt{kT}\,(\tfrac{1}{2}N_e^2 + \tfrac{1}{2}N_e^2)$$
$$= \underline{A_{br}\sqrt{kT}\,N_e^2}$$

Case 2:

$N_D = N_T = \frac{46}{100} N_e$ $\qquad$ $N_O = \frac{1}{100} N_e$

$$P_{br} = A_{br} N_D N_e Z_D^2 \sqrt{kT} + A_{br} N_T N_e Z_T^2 \sqrt{kT} + A_{br} N_O N_e Z_O^2 \sqrt{kT}$$
$$= A_{br}\sqrt{kT}\left(\tfrac{46}{100}N_e^2 + \tfrac{46}{100}N_e^2 + \tfrac{1}{100}N_e^2(8)^2\right)$$
$$P_{br} = A_{br}\sqrt{kT}\left(\tfrac{39}{25}N_e^2\right)$$
$$P_{br} = 1.56\, A_{br}\sqrt{kT}\, N_e^2$$

$$\text{POWER INCREASE} = \frac{1.56\, \cancel{A_{br}\sqrt{kT}}\, N_e^2}{\cancel{A_{br}\sqrt{kT}}\, N_e^2} = \boxed{1.56 \text{ INCREASE}}$$

$\boxed{3.5}$

$$\underline{P_{fu} = R_{fu} Q_{fu}}$$

FOR 2keV $\qquad$ $N_D = N_T = \frac{1}{2} N_e$

$$R_{fu} = N_D N_T \underbrace{\langle \sigma v \rangle_{ab}}_{\text{APENDIX C}}$$
$$= (\tfrac{1}{2}N_e)^2 \langle \sigma v \rangle_{ab}$$
$$= \tfrac{1}{4}(10^{20})^2(2.83\times10^{-25}) \leftarrow \text{FROM APENDIX C}$$
$$R_{fu} = 7.075\times10^{14}\ \frac{1}{m^3 s}$$
$$P_{fu} = 7.075\times10^{14}\ \frac{1}{m^3 s}(17.6\ \text{MeV})$$
$$\underline{P_{fu} = 1.2452\times10^{16}\ \frac{\text{MeV}}{m^3 \cdot s}}$$

$$P_{br} = A_{br} N_D N_e Z^2 \sqrt{kT} + A_{br} N_T N_e Z^2 \sqrt{kT}$$
$$= (1.6\times10^{-38})(\sqrt{2000}\ \text{eV})\left[(\tfrac{1}{2}10^{20})(10^{20})(1^2)\right]\cdot(2)$$
$$= 7155.418\ \frac{J}{m^3 \cdot s}$$
$$= 4.466\times10^{16}\ \frac{\text{MeV}}{m^3 \cdot s}$$

$$\frac{P_{br}}{P_{fu}} = \frac{4.466\times10^{16}}{1.2452\times10^{16}} = 3.587$$

(NEXT PAGE) →

3.5 CONTD

20 keV

$R_{fu} = \frac{1}{4}(10^{20})^2(4.31\times10^{-22}) = 1.0775\times10^{18}$

$P_{fu} = (1.0775\times10^{18})(17.6) = 1.896\times10^{19}\ \frac{MeV}{m^2 \cdot s}$

$P_{br} = (1.6\times10^{-38})(\sqrt{20{,}000})\left[(\tfrac{1}{2}10^{20})(10^{20})(1^2)\right]\cdot 2$

$P_{br} = 22627.4\ \frac{J}{m^3 s}$

$P_{br} = 1.41227\times10^{17}\ \frac{MeV}{m^3 \cdot s}$

$\boxed{\frac{P_{br}}{P_{fu}} = 0.00745}$

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$\boxed{3.6}$ (PLOT ON NEXT PAGE)

a) THE TWO INTERSECT AT ABOUT 4.75 keV

$KT = 4.75\ keV$

$T = \frac{4.75\ keV}{8.6117\times10^{-5}\ eV\cdot K^{-1}} \cdot \frac{1000\ eV}{1\ keV}$

$\boxed{T \approx 55\times10^6\ K}$

b) $d + t \rightarrow n + \underline{\alpha}$

$f_{c,dt} = \frac{3.5}{17.6} \approx 0.20$

$f_{c,dt}\ P_{dt}(T^*_{ign}) = P_{br}(T^*_{ign})$

$\frac{P_{dt}}{P_{br}} = 5$

∴ ON GRAPH, ESTIMATE WHERE P_{dt} IS ABOUT 5 TIMES GREATER THAN P_{br}

THIS HAPPENS AT ABOUT $T_{ign} \approx 9$ keV

THIS MEANS THAT THE ACTUAL ENERGY INPUT IS 9keV AND NOT 4.75 keV IN ORDER TO KEEP THE PLASMA POWER BALANCE

3.6

50:50% D-T fusion plasma

$N_e = 10^{14}\ cm^{-3}$ $\quad N_D = N_T = \frac{1}{2} N_e$ $\quad Z_D = Z_T = 1$

$P_{br} = A_{br}\sqrt{kT}\,(N_e)\left(\frac{1}{2}N_e + \frac{1}{2}N_e\right)(Z_T^2 + Z_D^2)$

$P_{br} = A_{br} N_e^2 \sqrt{kT} = 160\sqrt{kT}$ $\quad [W m^{-3}]$

$P_{fu} = \langle \sigma v \rangle N_i N_e Q_{fu}$

$= \langle \sigma v \rangle N_e^2 (17.6 MeV) \times 1.6022 \times 10^{-19}$

$= 2.81987 \times 10^{28} \langle \sigma v \rangle$ $\quad [W m^{-3}]$

just including power allocated to alphas (α s)

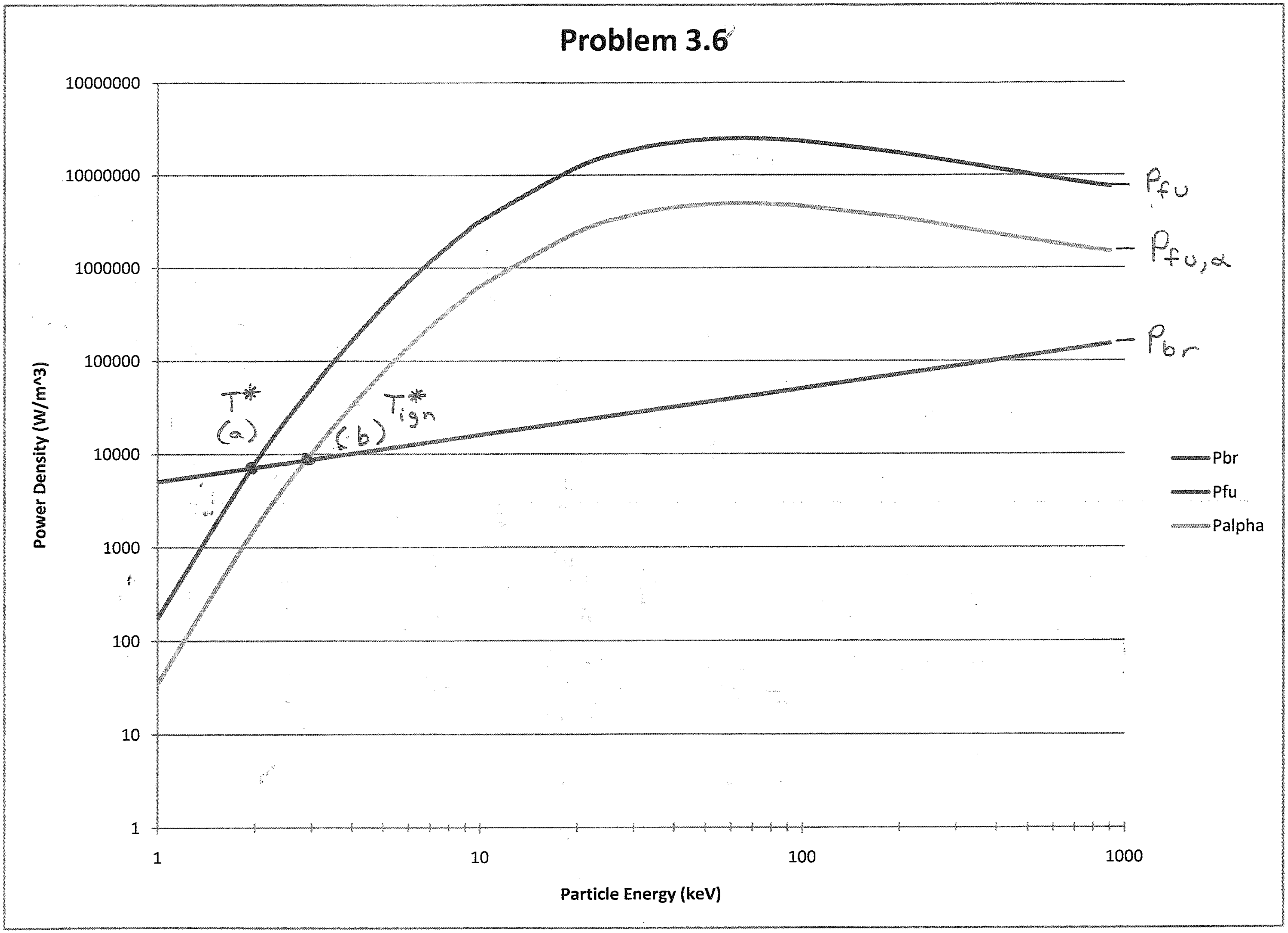
$P_{fu} = \langle \sigma v \rangle N_e^2 Q_{fu\alpha}$ $\quad Q_{fu\alpha} = 3.5 MeV$

$= 5.6077 \times 10^{28} \langle \sigma v \rangle$ $\quad [W m^{-3}]$

(a) ideal temperature (T^*) is labeled as point (a) on graph

(b) T^*_{ign} is labeled as point (b) on graph

This condition means that the T^*_{ign} is quite a bit larger than T^*. A higher temperature has to be balanced with losses due to Bremmsstrahlung radiation.


Problem 3.6
10000000
10000000
1000000
100000
10000
1000
100
10
1
Power Density (W/m^3)
1
10
100
1000
Particle Energy (keV)
Pbr
Pfu
Palpha
P_{fu}
$P_{fu,\alpha}$
P_{br}
T^*
(a)
(b)
T^*_{ign}