HW 7 Due Mar. 18

1) Derive 13.2 (G&L) from text book 6.25.Hint – as discussed in class, to convert from the ***v***x***B*** terms to electrical current $\boldsymbol{j}$, it is necessary to write the equations for both ions and electrons, then apply the definition of electrical current as the difference between flow of positive charge and negative charge.

2) Equation 13.2 (G&L) with the time derivative set to zero represents the steady state condition for pressure balance in a plasma. (Note this is used as the equilibrium condition for MHD equations. So that if electric field which does not vary with time and for the magnetic field lines of force are straight and parallel this reduces to the simple pressure balance we used earlier as given in our test as 4.11b or G&L in sec 13.5. Hint – to do this, first apply Maxwell's $\nabla \times \boldsymbol{B}$ equation to eliminate $\boldsymbol{j}$ and then use the vector expansion equation for $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}$ and note that $(\boldsymbol{B} \cdot \nabla)\boldsymbol{B}$ will be zero.

3. Starting with the equilibrium condition for p_0, $\mathbf{j_0}$, $\mathbf{B_0}$ given below 13.5 in G&L,

$$0 = -\nabla p_0 + \frac{1}{c}(\mathbf{j}_0 \times \mathbf{B}_0)$$

Show that linear perturbations of the parameters lead to (13.8).

4. The variational method, (13.12) of G&L provides considerable physical insight into stability. Instabilities in the pinch are often classified as either "current driven" or "pressure driven" (or both) depending on the driving force in the perturbed energy term δW. Based on the example in appendix A of G&L, identify which terms in δW (13.17-13.18) fall into these classifications.

Based on these thoughts, discuss the differences in stability between the Tokamak and Stellarator. [Hint- the stellarator achieves B field shear by using a "figure eight" shape while the Tokamak uses a "transformer action".] Which do you think should be more stable?

5. Consider the stability diagram of Fig 13.2 of G&L. (Assumes the plasma current is only on plasma surface)

a) The vertical axis is $b_i \equiv B_z(int)/B_\theta$. Show that if B_z is confined to the plasma ($B_z(ext) = 0$), $b_i = 1 - \beta$, where β is the "plasma β" or $\beta = 2nkT/(\frac{B_\theta^2}{2\mu})$, B_θ=field at exterior plasma surface.

b) Sketch a stability diagram for b_i^2 vs. the pinch ratio k for the pinch of a) where $B_z(ext) = 0$. Use data from Fig 13.2, indicate the region of stability against m=0, 1 on this sketch.

c) Optimum operation of the pinch would be in the stable region with the maximum fusion power density. Identify a point on the sketch of b) for such operation and indicate the reason for your choice.

6. a) Verify that derivation of Eq (13.51) of G&L starting with the normal mode equation of (13.39).

b) Verify that this equation is a form of Bessel's equation by identifying the general form for such equation (may need to refer to your old differential equ text?)

7. Assume the plasma current flows along a line on the axis of the cylindrical pinch described by the equations in Sec 13.29 of G&L. Obtain an expression for the shear vs. radius $\mu(r)$. Identify the change in shear values at r=0 to r=a=outer radius. Does this make sense? Discuss.