1. Discuss pressure balance in IEC. Set up an expression for it and comment on how mechanical force is transmitted to outside world. (Note $T_i >> T_e$) Is the mechanical structure required easier (or harder) to engineer than in a Tokamak? (Need to state what structure is required in the Tokamak)

The Inertial-electrostatic confinement (IEC) involves the creation of **deep electrostatic potential wells** within a plasma in order to accelerate ions to energies sufficient for fusion reactions to occur and to keep the ions confined. The kinetic pressure (of high energy ions) is balanced by the electrical force between ions and the biased grid, which tends to prevent the ions from escaping from the grid sphere.

$$p_i = N_i kT = \frac{\varepsilon_0 E^2}{2}$$

A mechanical force is transmitted to the grid through the electrical field.

For Tokamak, its coil wall also bears a high mechanical force due to the kinetic pressure, but transmitted through magnetic field. The pressure on the wall could be as high as 100 atmosphere pressure. Considering the coil is composed of super-conducting material which is very brittle, the supporting mechanical structure has to be arranged around the coil to prevent the coil from blowing up.

Since IEC only needs to balance the ion pressure (electron pressure is low and negligible), it is relatively easier to satisfy than in Tokamak where both ion and electron pressure need to be considered.

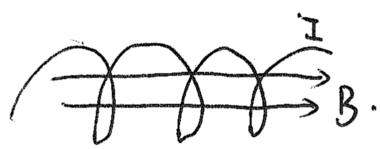
2. As you already know, it is a z-pinch experiment.

$\Rightarrow$ ~~The~~ A big capacitor bank is charged to high voltages, and then the breakdown happens and lasts for a very short time, which induces large current.

$\Rightarrow$ For z-pinch, $I^2 = 200NkT$

It assumes β as 1.

$\Rightarrow$ For θ-pinch.



$B_z = \mu_0 \tilde{N} I$ — $\tilde{N}$ is the number of turns per unit length.

$\frac{B_z^2}{2\mu_0} = P_{kinetic} = nkT$, n is the particle density ($\downarrow$ e^- and ions).

$\frac{\mu_0 (\tilde{N} I)^2}{2} = nkT$. — This is the equation in S.I. units.

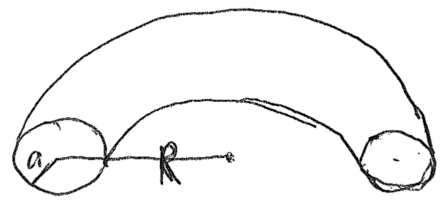
You can also try to change it in CGS units as done in the G&L book.

3. Plasma-wall interactions.

$n = 10^{14}\ cm^{-3} = 10^{20}\ m^{-3}$, 20 keV D-T. $\tau_c = 1s$

torus $R = 6\ m$, $a = 2.0\ m$.

i) power density by escaping particles (D, T)



Total number of D-T particles N_{total}.

$= $ volume of torus $\times$ density

$= \pi a^2 \cdot 2\pi R \cdot n$

$= \pi \times 2^2 \times 2\pi \times 6 \times 10^{20} = 4.74 \times 10^{22}$ (particles)

power density due to D.T.

$$= \frac{N_{total} \times \bar{E}}{\text{Area of torus} \times \text{confine time}}, \quad \text{with } \bar{E} = \frac{3}{2} kT$$

$$= \frac{4.74 \times 10^{22} \times \frac{3}{2} \times 20 \times 10^{3} \times 1.6 \times 10^{-19}}{2\pi a \times 2\pi R \times 1(s)}$$

$= 4.80 \times 10^{5}\ J/m^2 \cdot s = 4.80 \times 10^{5}\ W/m^2$

$$\text{Erosion rate} = \frac{\text{Thickness}}{\text{Confine time} \sim (1s)}$$

$$= \frac{\text{Total Fe atom number knocked out}}{\cancel{N_A}} \times M_{Fe} \cdot \frac{1}{\rho_{Fe} \cdot S_{torus}} \cdot \frac{1}{\tau_c}$$

$\tau_c \downarrow$ 1s

$$= \frac{\frac{1}{5} \times N_{total} \times 55.845 \times 10^{-3} (kg)}{6.02 \times 10^{23}} \cdot \frac{1}{7.874 \times 10^{3} (kg/m^3) \cdot \underbrace{2\pi a \cdot 2\pi R}_{(m^2)} \times 1s}$$

$= 2.36 \times 10^{-10}\ m/s$

$= \cancel{1.16 \times 10^{1}}\ \cancel{11.59\ m/y}$

$= \cancel{1.16 \times 10^{4}\ mm/y}\ 7.43\ mm/y$.

This could be a serious problem.

① 7mm/year is a huge damage to the wall, Replacing the chamber every one or two years costs too much.

② Fe ~~ions~~ from the cold chamber wall will reduce the plasma temperature, consuming a lot of energy and causing problem for plasma maintaining

③ Fe impurity cause much more Bremsstrahlung radiation, which is an energy loss.

UIUC group is trying to use Liquid Lithium wall. instead of Fe wall. When D/T hits the Li, they are absorbed rather than sputter the Li out into the plasma.

ii). Without Fe ions. $(N_{i0} = N_{D0} + N_{T0})$ ~ initial case without Fe.

$$P_{br} = N_{i0} N_e z^2 \sqrt{kT}\, A_{br} \qquad (kT \text{ in unit of eV})$$

$$= N_{i0}^2 \sqrt{20000} \cdot A_{br} = (10^{20})^2 \times \sqrt{20000} \times 1.6 \times 10^{-38} = 2.26 \times 10^4 \ W/m^3$$

$$P_{cyc} = A_{cyc} N_e B^2 \psi \cdot kT_e.$$

B is the internal magnetic field, not external B_0

Use $\dfrac{B_0^2}{2\mu_0} = \dfrac{B^2}{2\mu_0} + \underbrace{N_{i0} kT + N_{e0} kT}_{2 N_{i0} kT}$ in unit of J

$$\beta = \frac{B_0^2/2\mu_0}{2 N_{i0} kT}$$

$$\Rightarrow B^2 = 2\mu_0 \frac{1-\beta}{\beta} (2 N_{i0} kT) = 2 \times 1.2566 \times 10^{-6} \frac{1-0.3}{0.3} \times 2 \cdot N_{i0} \times 20000 \times 1.6 \times 10^{-19}$$

$$= 3.75 \times 10^{-20} N_{i0} \ (T^2).$$

$$\Rightarrow P_{cyc} = \cancel{A_{cyc} \cdot 4\mu \cdot \frac{1-\beta}{\beta} N_{i0}^2 (kT)^2 \psi}$$

$$A_{cyc} N_{i0} \cdot B^2 \cdot \psi \ kT \quad \text{in unit of eV.}$$

$$= 6.3 \times 10^{-20} \times N_{i0}^2 \times 3.75 \times 10^{-20} \times 10^{-2} \cdot 20000$$

$$= 4.73 \times 10^3 \ W/m^3$$

~~With A Fe ion (Fe^{26+}). N_{Fe} = A~~

Then consider the case with $\underset{(Fe^{+26})}{\underline{\text{Fe ion}}}$ impurity: $\frac{N_{Fe}}{N_{i0}} = x$.

$$N_e = N_{i0} + 26\,x \cdot N_{i0} \qquad Z = 26 \text{ for Fe ion.}$$

$$P'_{br} = N_{i0} N_e \sqrt{kT} A_{br} + N_{Fe} \cdot N_e \sqrt{kT} \cdot 26^2 \cdot A_{br}$$

$$= \left[N_{i0}^2 (1 + 26x) + N_{i0}^2 x \cdot (1+26x) \times 26^2 \right] \sqrt{kT} \cdot A_{br}$$

$$= \underbrace{2.26\times10^4}_{P_{br}\text{ for 1st case}} (1 + 702x + \cancel{1376}\ 17576x^2)$$

While for P_{cyc}.

$$B^2 = 2\mu_0 \frac{1-\beta}{\beta} \cdot (N_i kT + N_e kT)$$

$$= 2\mu_0 \frac{1-\beta}{\beta} \cdot N_{i0} kT (\underbrace{1 + x + 1 + 26x}_{\to 2 + 27x})$$

$$= 2\mu_0 \frac{1-\beta}{\beta} 2 N_{i0} kT (1 + 13.5x)$$

$$P'_{cyc} = \underbrace{4.73\times10^3}_{P_{cyc}\text{ for 1st case}} (1 + 13.5x)(1+26x)$$

Let $P'_{br} + P'_{cyc} = 2(P_{br} + P_{cyc})$

$$\Rightarrow 2.26\times10^4 (1 + 702x + 17576x^2) + 4.73\times10^3 (1+13.5x)(1+26x)$$

$$= 2 \times (2.26\times10^4 + 4.73\times10^3).$$

$$\Rightarrow x = 1.64\times10^{-3}$$

$$= 0.164\%$$

As you can see here, only a very small fraction of Fe impurity as low as 0.164% could cause the radiation to double.

Radiation power density

$$= \frac{(P_{br} + P_{cyc}) \cdot \text{Volume}}{\text{Area}} = \frac{(2.26\times10^4 + 4.73\times10^3) \times \pi a^2 2\pi R}{2\pi a \cdot 2\pi R}$$

$$= 2.73\times10^4 \ W/m^2$$

Neutron power density.

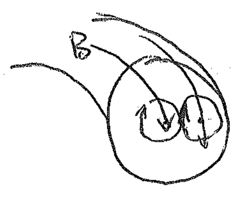
$$P_{nt} = \frac{\frac{1}{4} N_i^2 \langle \sigma v \rangle E_n \cdot V}{\text{Area}.} \qquad E_n = 14.1\ \text{MeV}. \qquad \langle \sigma v \rangle_{kT=20keV} = 4.31 \times 10^{-22}\ m^3 s^{-1}$$

$$= \frac{\frac{1}{4} \times (10^{20})^2 \times 4.31 \times 10^{-22} \times 14.1 \times 10^6 \times 1.6 \times 10^{-19} \times \pi a^2 \cdot 2\pi R}{2\pi a \cdot 2\pi R}.$$

$$= 2.43 \times 10^6\ W/m^2$$

So. ~~Pract~~ Wall loads from radiation, lost particles and neutron. are 2.73×10^4, 4.80×10^5 and 2.43×10^6 W/m² respectively.

4.



$\alpha - 3.5$ MeV $\quad N_i = N_e = 10^{14}\ cm^{-3} = 10^{20}\ m^{-3}$

To satisfy $r_{g\alpha} < a$, need to find $r_{g\alpha}$ first.

$$r_{g\alpha} = \frac{m_\alpha v}{qB}$$

Internal B field $= B = \sqrt{\frac{2\mu_0 (1-\beta) 2 N_i kT}{\beta}}$

$$= \sqrt{\frac{2 \times 1.2566 \times 10^{-6} \times 0.7 \times 2 \times 10^{20} \times 20 \times 10^3 \times 1.6 \times 10^{-19}}{0.3}}$$

$$= 1.94\ T.$$

$m_\alpha = 6.6467 \times 10^{-27}$ kg

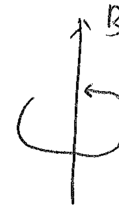
$$v = \sqrt{\frac{2E}{m_\alpha}} = \sqrt{\frac{2 \times 3.5 \times 1.6 \times 10^{-19}}{6.6467 \times 10^{-27}}} = 1.32 \times 10^7\ m/s$$

$$r_{g\alpha} = \frac{6.6467 \times 10^{-27} \times 1.32 \times 10^7}{2 \times 1.6 \times 10^{-19} \times 1.94} = 0.141\ m.$$

So, the minimum a (minor radius) should be 0.141 m.

5.7. $\mu = \dfrac{\frac{1}{2} m V_\perp^2}{B}$ (5.76)

Consider the case of an ion moving in B-field.



An ion with q as the charge and $V_\perp$ as the speed, the gyration radius $r_g = \dfrac{m V_\perp}{qB}$

It's equivalent to a current loop with.

$$I = \frac{q}{T} \sim \text{period of the gyration} \qquad T = \frac{2\pi r_g}{V_\perp}.$$

$$= \frac{q V_\perp}{2\pi r_g}$$

$$\cancel{= \frac{q V_\perp}{2\pi} \cdot \frac{qB}{m V_\perp}}$$

The area of the loop $A = \pi r_g^2$.

So, $I \cdot A = \dfrac{q V_\perp}{2\pi \cancel{r_g}} \cdot \pi r_g^{\cancel{2}}$

$$= \frac{q V_\perp}{2\pi} \cdot \pi \cdot \frac{m V_\perp}{qB}$$

$$= \frac{m V_\perp^2}{2B} = \mu.$$

The unit of $\mu = \dfrac{kg \cdot (m/s)^2}{T}$, while $T = \dfrac{kg}{A \cdot s^2}$

$$\Rightarrow \; = m^2 \cdot A.$$

That just accords with the unit of μ from $\mu = I \cdot A$.

5.9. DT 50% : 50%, $T_i = T_e$, $\beta = 0.2$, $\psi = 10^{-3}$.

$$P_{br} + P_{cyc}^{net} = f_{c,dt} P_{dt}.$$

① $P_{br} = A_{br} \cdot N_i N_e z^2 \sqrt{kT}$

$$= A_{br} \cdot \cancel{\#} N_e^2 \cdot \sqrt{kT} \cancel{\#\#\#\#} = 1.6\times10^{-38} \cdot N_e^2 \sqrt{kT}$$

kT in J

② $P_{cyc}^{net} = A_{cyc} N_e \cdot B^2 \cdot kT_e \psi$ $\left[B^2 = B_0^2 (1-\beta)\right] = \frac{2\mu_0 \cdot 2 N_e kT (1-\beta)}{\beta}$

internal external

$$= A_{cyc} \cdot \frac{N_e^2 \cdot (kT)^2 \cdot 10^{-3} \cdot 1.6\times10^{-19} \cdot (1-0.2)}{1.2566\times10^{-6} \times 0.2}$$

$$= \frac{4 N_e \cdot kT \cdot 1.6\times10^{-19}}{(1.2566\times10^{-6})^{-1}} \cdot \frac{(1-0.2)}{0.2}.$$

[kT in eV]

$$= A_{cyc} \times \cancel{5.093\times10^{-16}}\ 3.217\times10^{-27} \cdot N_e^2 (kT)^2.$$

$$= \cancel{3.209\times10^{-35}}\ 2.027\times10^{-46}\ N_e^2 (kT)^2.$$

③ $f_{c,dt} P_{dt} = \frac{1}{5} \times P_{dt}$

$$= 0.2 \cdot \frac{1}{4} N_e^2 \langle\sigma v\rangle_{dt} \cdot 17.6\times10^6 \times 1.6\times10^{-19}$$

$$= 1.408\times10^{-13} \cdot \langle\sigma v\rangle_{dt} \cdot N_e^2.$$

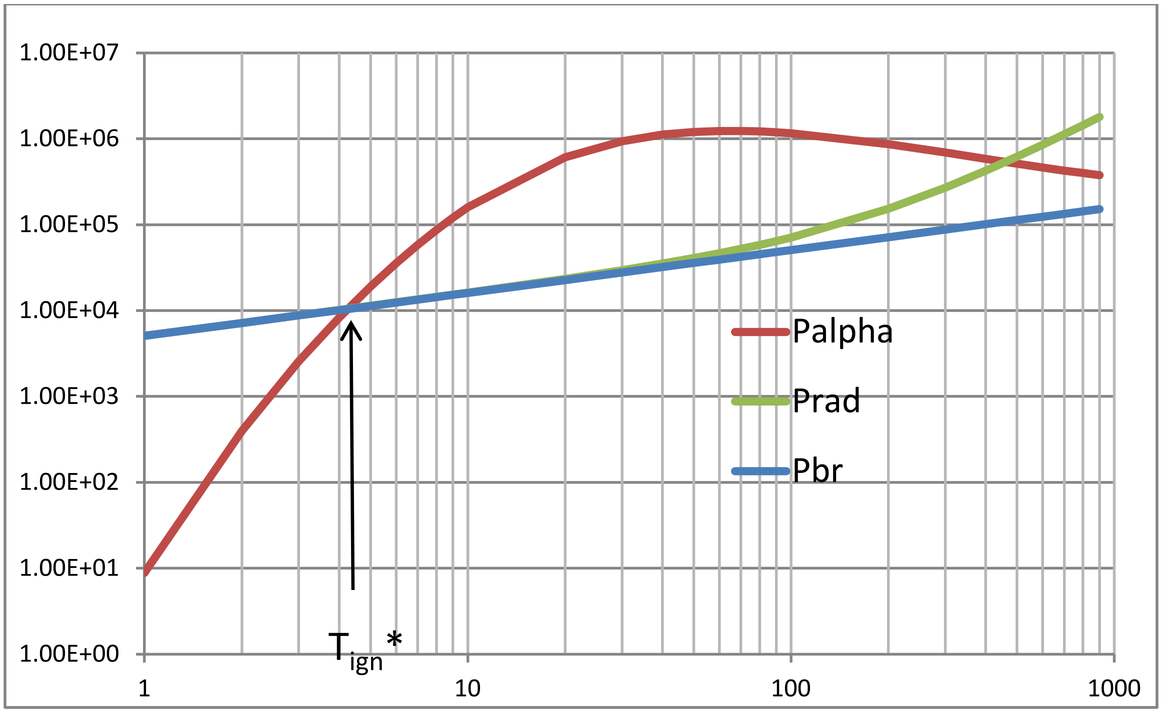
$\Rightarrow$ $1.6\times10^{-38} N_e^2 \sqrt{kT} + 3.209\times10^{-35} N_e^2 (kT)^2 = 1.408\times10^{-13} \langle\sigma v\rangle_{dt} N_e^2$

$\Rightarrow$ obviously, T_{ign}^* doesn't depend on N_e. (also depending on kT.)

Let $N_e = 10^{14}\ cm^{-3} = 10^{20}\ m^{-3}$

$$\underbrace{1.6\times10^{2}\sqrt{kT}}_{P_{br}} + \underbrace{\cancel{3.209\times10^{5}}\ 2.027\times10^{-6} (kT)^2}_{P_{rad}} = \underbrace{1.408\times10^{27} \langle\sigma v\rangle_{dt}}_{P_\alpha\ (P_{alpha})}$$

Use graph attached, $T_{ign}^* \hat{=} 4.28$ keV; slightly increased comparing to problem 3.6.



1.00E+07
1.00E+06
1.00E+05
1.00E+04
1.00E+03
1.00E+02
1.00E+01
1.00E+00
1
10
100
1000
Palpha
Prad
Pbr
T_{ign}*

EXTRA CREDIT

5.4

$$F = \underbrace{q\frac{|\nabla B|}{B_0}\left[\text{sgn}(q)\frac{\omega_g}{2\pi}\right.}_{①}\underbrace{\left.\int_0^{2\pi/\omega_g}\frac{V_\perp^2}{\omega_g}B_0\cos^2(\omega_g t+\phi)\,dt\right]}_{②}\hat{y}$$

② $\int_0^{2\pi/\omega_g}\frac{V_\perp^2}{\omega_g}B_0 \frac{1+\cos(2\omega_g+2\phi)dt}{2}$

$$\frac{V_\perp^2}{\omega_g}B_0\int_0^{2\pi/\omega_g}\frac{1+\cos(2\omega_g+2\phi)\,dt}{2}$$

$$\frac{V_\perp^2}{\omega_g}B_0\left(\frac{t}{2}+\frac{\sin(2\omega_g t+2\phi)}{2\omega_g\cdot 2}\right)\Bigg|_0^{2\pi/\omega_g}$$

$$\frac{V_\perp^2}{\omega_g}B_0\left(\frac{\pi}{\omega_g}+\frac{\sin(4\pi t+2\phi)}{4\omega_g}-0-\frac{\sin(2\phi)}{4\omega_g}\right)$$

① $F = q\frac{(\nabla B)}{B_0}\left(-\text{sign}(q)\frac{\omega_g}{2\pi}\right)\frac{V_\perp^2}{\omega_g}B_0\left(\frac{\pi}{\omega_g}\right)\hat{y}$

$$\vec{F} = q\frac{|\nabla B|}{B_0}\left(-\text{sign}(q)\frac{\omega_g}{2\pi}\right)\frac{V_\perp^2}{\omega_g}B_0\pi\,\hat{y}$$

$$\vec{F} = q\frac{|\nabla B|}{B_0}\frac{V_\perp^2}{\omega_g^2}B_0\pi - \text{sign}(q)\frac{\omega_g}{2\pi}\hat{j}$$

$$\vec{F} = \left[\frac{V_\perp^2}{2\omega_g}q|\nabla B|\cdot\text{sign}(q)\right]\hat{y}$$

$$\boxed{\vec{F} = -|q|\frac{V_\perp^2}{2\omega_g}\nabla B}$$

5.5.

$$\frac{dV_{gc,z}}{dt} = \text{sign}(q)\, w_g \frac{V_\parallel V_\perp}{R} t \cos(w_g t) \qquad \text{——— (5.56 c)}$$

$$dV_{gc,z} = \text{sign}(q)\, w_g \frac{V_\parallel V_\perp}{R} t \cos(w_g t)\, dt$$

$$\int_0^{V(t)} dV_{gc,z} = \int_0^t \text{sign}(q)\, w_g \frac{V_\parallel V_\perp}{R} t' \cos(w_g t')\, dt'$$

$$\Rightarrow V_{gc,z}(t) - V_{gc,z}(0) = \text{sign}(q)\, w_g \frac{V_\parallel V_\perp}{R} \cdot \left[\frac{\cos(t' w_g)}{w_g^2} + \frac{t' \sin(w_g t')}{w_g} \right] \Bigg|_{t'=0}^{t'=t}$$

$$= \text{sign}(q)\, w_g \frac{V_\parallel V_\perp}{R} \left[\frac{\cos(t w_g) - 1}{w_g^2} + \frac{t \sin(w_g t)}{w_g} \right]$$

Integrate over a gyration period τ. (so $w_g \tau = 2\pi$)

$$\overline{V_{gc,z}} - V_{gc,z}(0) = \int_0^\tau \text{sign}(q)\, w_g \frac{V_\parallel V_\perp}{R} \left[\frac{\cos(w_g t) - 1}{w_g^2} + \frac{t \sin(w_g t)}{w_g} \right]$$

$$= \text{sign}(q)\, w_g \frac{V_\parallel V_\perp}{R} \left[\underbrace{\frac{\frac{1}{w_g} \sin(w_g t)}{w_g^2} \Bigg|_0^\tau}_{=0} - \frac{t}{w_g^2} \Bigg|_0^\tau + \frac{t \cdot (-\cos w_g t)}{w_g^2} \Bigg|_0^\tau + \underbrace{\frac{\sin(w_g t)}{w_g^3} \Bigg|_0^\tau}_{=0} \right]$$

$$= \text{sign}(q)\, w_g \frac{V_\parallel V_\perp}{R} \cdot \left(-\frac{2\tau}{w_g^2} \right) < 0.$$

for $q > 0$, The average $\overline{V_{gc,z}}$ is smaller than ~~the initial~~ speed, meaning it's drifting anti-parallel to the $\vec{B}$-field, in the "$-z$" direction

And the drift depends on the $V_\parallel$ value.

for $q < 0$, not sure if it is the case.