Exam 2.

1. a. $F_{\parallel} = \frac{1}{2} q V_\theta r \frac{\partial B_z}{\partial z}$.

at $r=0$ $\vec{F_\parallel} = -\frac{1}{2} \frac{mV_\perp^2}{B} \cdot \nabla_\parallel B$, so $\vec{F_\parallel}$ is in the opposite direction of $\vec{V_\parallel}$ if particles move towards "magnetic field opening", and thus reduces the $\vec{V_\parallel}$.

The reflection will be when $\vec{V_\parallel}$ reduces to zero.

b. $F_\parallel = -\frac{1}{2} \frac{mV_\perp^2}{B} \nabla_\parallel B$ is the same for ions and electrons at any point.

$$\int F_\parallel ds = W = \Delta E.$$

Since the initial E is also the same, so ions and eletrons will travel for similar distance before reflected.

c. $\vec{F_\parallel} = -\frac{1}{2} \frac{mV_\perp^2}{B} \nabla_\parallel B$

$\Downarrow$ P78, 79. Equ. 5.73 $\rightarrow$ 5.80

$\frac{d}{dt}(\mu) = 0.$

It requires the B. gradient could not be too large, because of the assumption used in Equ. 5.65

$$B_r = -\frac{1}{r}\int_0^r r' \frac{\partial B_z}{\partial z} dr' \approx -\frac{r}{2} \frac{\partial B_z}{\partial z}.$$

Specifically, $\frac{\partial B_z}{\partial z}$ should be constant during one gyration cycle.

$$\left.\begin{aligned} &\frac{r_g}{B} \nabla_\parallel B \ll 1. \\ \text{and } &\frac{1}{\omega_g B} \frac{dB}{dt} \ll 1. \end{aligned}\right\}$$

2. a. Assumptions:

① Collisions frequently that $\vec{v}$ distribution of particles is isotropic or say, $\lambda \to 0$.

② Heat flow is inhibited, system adiabatic.

③ plasma T is high → plasma a perfect conductor.

It these break down, particle kinetic description is needed.

b. plasma is assumed as perfectly conducting. $\sigma \to \infty$

$$E + \frac{1}{c}(\vec{v} \times \vec{B}) = 0$$

$\longrightarrow$ Ohm's law.

c. $\frac{dp}{dt} + \gamma p (\nabla \cdot \vec{v}) = 0$

and $\frac{d\rho}{dt} + \rho \nabla \cdot \vec{v} = 0$ Eq. 13.1 of G&L

$$\frac{1}{p}\frac{dp}{dt} = \frac{\gamma}{\rho} \cdot \frac{d\rho}{dt}$$

Integrating ⇒ $p = c \cdot \rho^{\gamma}$

adiabatic behavior of ideal gas.

(cf. 13.1 ~ 13.3 in G&L)

d. $D_\perp = \frac{r_g^2}{\tau_c} = r_g^2 \, n \langle \sigma v \rangle_s$ ⟶ $\langle \sigma v \rangle_{si} = \langle \sigma v \rangle_{si\text{-}i} + \cancel{\langle \sigma v \rangle_{si\text{-}e}}$

collided by ions. (collided by e neglect.)

$$\frac{D_{\perp i}}{D_{\perp e}} = \frac{r_{gi}^2 \langle \sigma v \rangle_{si}}{r_{ge}^2 \langle \sigma v \rangle_{se}}$$

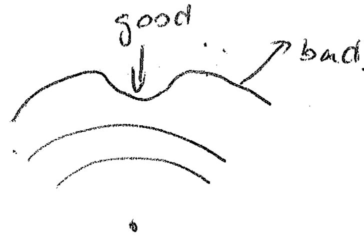
$\langle \sigma v \rangle_{se} = \langle \sigma v \rangle_{se\text{-}e} + \langle \sigma v \rangle_{se\text{-}i}$

usually not negible, but for simplification, not considered here.

$$\approx \frac{(v_i m_i)^2}{(v_e m_e)^2} \cdot \frac{\frac{v_i}{(m_i v_i^2)^2}}{\frac{v_e}{(m_e v_e^2)^2}}$$

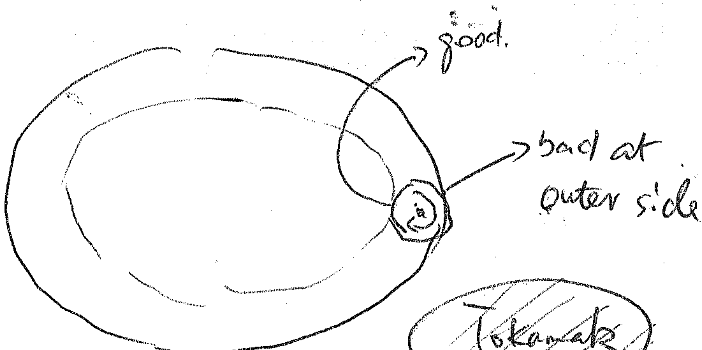
$$\approx \frac{v_e}{v_i} \gg 1.$$

3\. (a) The bad curvature causes "flute-type" instability. ($\vec{E}\times\vec{B}$ drift).



Pg. 146 or P485 in G&L

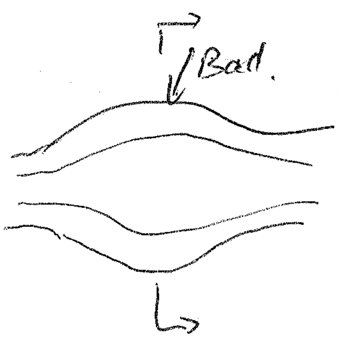
(b)



reduced by introduced B_p, and thus shear

Tokamak

Mirror



Use "minimum B", Ioffe bars, cusp or Yin-Yang coil. etc. to reduce the instability.

c) $F(\xi) = -\rho_0 \frac{d^2\xi}{dt}$

(ξ): displacement variable vector → any pertubations of plasma in any direction.

For stable system, ~~$\vec{F}(\xi)$ is alway opposite to $\vec{\xi}$, or~~

$\delta W > 0$ is required, where $\delta W = \int \xi \cdot F\xi \, d^3x$ is the change of potential energy resulting from the displacement ξ.

(d). $\delta W > 0$ means for every conceivable small perturbations, the potential energy is increased, by doing work on the system.

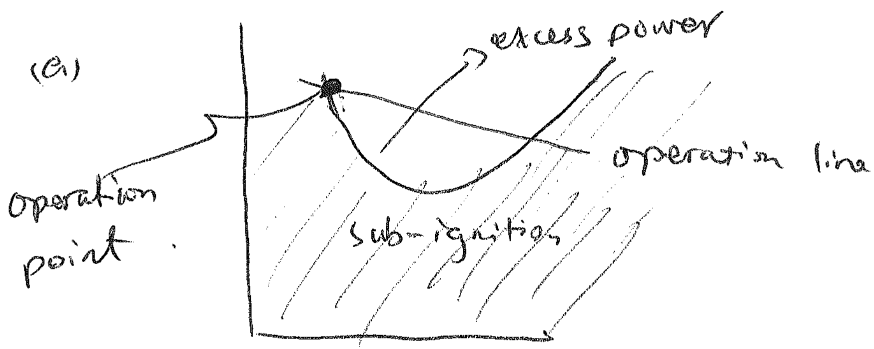
(e). The equation for f_r contains f_{r+1}. To accomodate the cutting off of this sequence, a physical approximation for f_{r+1} is needed in terms of $f_1, \dots f_r$ in order to arrive at a solvable set of equations.

For the $r=1$ case,

$$f_2(r_1 v_1, r_2 v_2, t) = f(1) f(2) + C(1,2) \text{ is needed.}$$

It physically represents the field force applied to the particle one due to the presence of other particles.

4. (a)



(b) Need to increase $n\tau$.

$$n\tau = k a^2 B_0^2 \sqrt{kT}$$

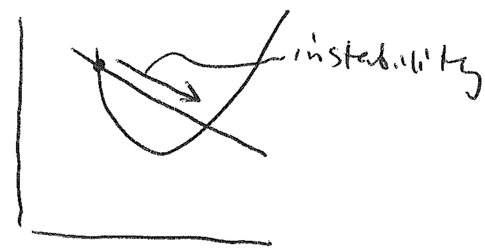
increase a, thus the size of tokamak.

or B_0, but it is limited at 13 T as the highest value achievable.

c. Probably no. Stellarator doesn't need current to create poloidal B field, so there won't be current-driven instabilities. But there should still be some pressure-driven ones.

But all of these are still being argued by Tokamak & Stellarator people..

d. Thermal instability: From the operation point shown ~~above~~ in $n\tau$-kT diagram the temperature of plasma tends to increase because of excess power generation.



This could be avoided by certain cooling methods, for example reduce the power density by injecting some impurity into plasma.

(e) τ_p as the the particle confine time, is the mean time of a particle stays in the confined volume.

$$\tau_p = \frac{N}{F}$$ leakage rate. $\sim$ ~~flux~~ to the chamber wall.

τ_E related w/ the energy ~~leak~~ leakage due to particles leakage.

$$\tau_E = \frac{E}{P}.$$

$\tau_E = \tau_p$ is every particle escape w/ the average energy $\frac{3}{2}kT$ but considering the higher energy partizles are much easier to leak, $\tau_E < \tau_p$.

(f). Neoclassical diffusion involves with trapped particles ;

i.e. particles ~~that~~ move following a banana orbit.

$$D = \frac{\Delta r_{trap}^2}{\tau_c}$$ instead of $D = \frac{r_g^2}{\tau_c}$ in classical case.

$\Delta r_{trap} > r_g$.

So Diffusion coefficient increases in neoclassical case and particles may escape more rapidly. $\Rightarrow$

Neoclassical diffusion sets the upper limit on losses.