NPRE 421 Take home exam

Due May 6 by 5 pm. Will be collected by Liang Meng and make sure sign your name in a provided sign sheet as a proof of turning it in. Also sign honor pledge that the work is your own and no help was received.

1. Matter and Energy Accounting

The carbon cycle is given by the overall equation (1.33). Are the energies of the individual particles produced (α, β^+, ν) fixed or variable? Explain your answer. Find their energies or possible range of energies.

2. Physical characteristics

Consider a beam of deuterons of energy E_b injected into a tritium target plasma (Maxwellian distribution with temperature T_t (cf. Fig 2.6). Set up (do not solve) the equation for calculation of sigma-v ($\langle\sigma v\rangle$) for the resulting fusion reaction. It is suggested that the background could be approximated as having a single temperature, T_t, i.e. with a delta-function energy distribution. Set up the expression for this approximation for $\langle\sigma v\rangle$. Does the approximated $\langle\sigma v\rangle$ over or underestimate to actual $\langle\sigma v\rangle$? Explain your reasoning.

3. Charged particle scattering

Compare (by calculation of their ratio) the total scattering cross section (Equ. 3.32) for deuterons and electrons in a 15 keV plasma. Discuss the physical explanation for your answer.

4. Radiation Emission.

Consider a D-T Tokamak plasma with $n=10^{14}$ cm^{-3}, B=6 Tesla, and 0.1 atom % Fe impurity). Find the plasma temperature, kT_p, where the Bremsstrahlung and Cyclotron emission loss rates are equal.

5. Consider the cylindrical plasma of problem 6.4. However assume that the source is from gaseous fuel “puffed” into the plasma. Thus the effective source is peaked at the plasma surface and can be approximated as

$$S(r)=1-r^2/a^2 \quad 0<r<a$$

Find the plasma density profile n(r). If this is a D-D plasma, set up and expression for the fusion power density profile, $P_f(r)$.

Part II (Same instructions as Part I. Turn in two parts together. Part II problems have double scoring weight as Part I.

6. Kinetic Treatment and Stability

The kinetic equation for a one particle distribution function is given by (6.32). Consider a plasma of ions and electrons. In this case electrons represent a second species which you may consider as a second set of particles so (6.32) will be written twice: once for ions and once for electrons.

a) Write out these two equations, including their closure by a cluster expansion. Assume i-e, e-i scattering can be neglected but include i-I and e-e collisions to represent the corresponding cluster expansion of (6.33 c).

b) Show that this set of equations reduces to the fluid equation (6.25) in the limit as the mean-free-length for collisions goes to zero.

c) Write out the full set of Maxwell equations including Poisson's equation, global neutrality and an "Ohm's Law" that must be combined with the two kinetic equations of part a.

d) What is needed for closure of this set of equations? Explain your answer explicitly using equations.

e) Show how Poisson's equation is related to the two kinetic equations of a), using explicit equations.

f) Show how a perturbation of these equations could be done to formulate an equation of motion for the displacement vector as done in ideal MHD stability theory.

7. Open magnetic system.

Consider a magnetic mirror fusion reactor with B_{max}=10 T and B_{min}=2 T. It is fueled with 50/50 D-^{3}He and achieves a plasma beta of ~0.9 (relative to the central volume plasma).

a) Develop an ignition line for this plasma on a nτ vs. kT diagram (cf to Fig 8.4 – but eliminating n in favor of beta as done in Exam 2.) [Note: retain both Bremsstrahlung and Cyclotron radiation losses for the D-^{3}He plasma.]

b) What is the ratio of minimum nτ for this D-^{3}He plasma vs traditional D-T? What is the ratio of temperatures where these minima occur?

c) Formulate the operating $n\tau$ line for the minor reactor and plot it on the figure for part a. Does an intersection occur? If not, what would you change in the reactor design to achieve an intersection?

d) Discuss the difference in kT for the mirror reactor vs that for a Tokamak such as in Exam 2. How does this difference affect the relative power densities and recalculating power for the two reactors?

e) "Advanced" mirror reactor concepts include the "bumpy" torus; the Elmo bumpy torus; the field reversed mirror, and the Tandem mirror. Sketch each indicating the location of magnets, injection units, divertors and other key features. Discuss advantages and disadvantages of each, indicating at least two advantages and two disadvantages of each.

8. Inertial confinement fusion.

a) Do problem 11.4 in the text.

b) Compare this result for case (b) of 11.4 with the pellet energy multiplication requirement of Eq (11.27). Assume that η_{fu}=1/3, η_{in}=1/3 and η_c=0.9. If Eq (11.27) is not satisfied by this target design, what changes would you propose to achieve the requirement?

c) Problem 11.4 does not include hot spot ignition. If that occurs such that its $\rho R \sim 0.3$ gm-cm^{-2}, estimate the increase in improvement in Q_{pp}. Do this by first reformulating Eq 11.27 to explicitly include the different in hot spot energy requirement vs. total compression energy. [Hint: for evaluation make reasonable estimates of hot spot vs. outer fuel temperatures upon compression].

d) Answer problem 11.6. Based on your answer, discuss the logic in going from the top two pellet designs in figure 11.3 to the "high gain ion beam pellet" at the bottom of the figure.

e) The Raleigh-Taylor instability is crucial to ICF performance, It can occur at the outer target surface and at the cold-hot spot plasma interface. Explain why it may occur at each location. Indicate at least two strategies to minimize the Raleigh-Taylor growth rate at each location.