Partial solutions to the midterm. (470)

The most "damaging" part of the midterm turns out to be Questions 3 and 4 of the open book part.

Open book Q 3

Depending on how one defines "recycled back once," one might define two different but similar answers.

1) The most simple and naive way is to write

\[ \frac{1}{\lambda} + \frac{1}{\lambda} \left( \frac{\lambda-1}{\lambda} \right) \]  

first pass used, second pass using the "first pass non-used"

This would give

\[ \frac{2\lambda-1}{\lambda^2} \]  

This result didn't consider the dilution effect of the recycle, and is therefore, not 100% right in the sense of asymptotic behavior when \( \lambda \to \infty \)

2) The addition in (1) needs to be weighted with the concentration difference.

\[ \rho \]
In general, we can find the overall expression by dimension analysis and asymptotic behavior.

(a) Note. \( \lambda \) is from \( 1 \to \infty \) so in general, the expression needs to be linear in \( \lambda \) in the numerator and quadratic in the denominator, actually, (2) fits in this demand nicely.

(b) check \( \frac{\lambda}{\lambda^2 - \lambda + 1} \). when \( \lambda \to \infty \) the expression

\[
\to \frac{1}{\lambda}
\]

fit our definition, when \( \lambda \to 1 \),

\[
\frac{\lambda}{\lambda^2 - \lambda + 1} \to 1
\]

Therefore, \( \frac{\lambda}{\lambda^2 - \lambda + 1} \) is a better result, because at larger, it gives \( \frac{1}{\lambda} \). While the \( \frac{2\lambda - 1}{\lambda^2} \) gives \( \frac{2}{\lambda} \).
\[ \frac{2\lambda - 1}{\lambda^2} \] is more applicable to the case the fuel is cascaded to two fuel cells. While \[ \frac{\lambda}{\lambda^2 - \lambda + 1} \] is better with the fuel circulating back into the inlets again, which should not matter with recirculation when the \[ \lambda \] is big (utilization is small).

Open book Q4

Note slope = \[ \frac{RT}{\Delta nF} \] but the intercept of the line on the \( I \cdot \ln|j| \) axis is \( \ln j_0 \) only!

The number \( \frac{RT}{\Delta nF} \ln(j_0) \) is the intercept on the \( \eta(v) \) axis which really depends on where one takes the origin of the \( \ln|j| \).

So the intercept is \( \ln(j_0) \) but not \( \frac{RT}{\Delta nF} \ln(j_0) \)

The \( T \) dependence of \( \frac{RT}{\Delta nF} \) is clear there.

As for \( \ln j_0 \), \( j_0 = nFc_{\text{ref}} f e^{-\Delta G^+ / RT} \)

\[ \frac{\partial \ln j_0}{\partial T} = \frac{+\Delta G^+}{RT} \cdot \frac{H}{T^2} \]
The pressure dependence of the both comes from the Nernst equation, in that reactivation barrier, depends on the concentration and pressure.

The change in $\Delta G^+$ is given by $\frac{RT}{nF} \ln \left( \frac{P}{P_0} \right)$.

Therefore \[ \frac{d\Delta G^+}{dP} = \frac{d}{dP} \left( \frac{RT}{nF} \ln \left( \frac{P}{P_0} \right) \right) = \frac{RT}{nF} \cdot \frac{1}{P} \left( \frac{P}{P_0} \right) \]

Very similarly, the slope is given by

\[ b = \frac{RT}{\alpha nF} - \frac{RT}{nF} \left[ \ln \left( \frac{P}{P_{\text{std}}} \right) \right] \]

Differentiate the above vs. $P$ should give the $P$ dependence.

The End.