Partial solutions to the Midterm (470)

The most "damaging" part of the midterm turns out to be Questions 3 and 4 of the open book part.

Open book Q 3

Depending on how one defines "recycled back once" one might define two different but similar answers.

① The most simple and naive way is to write

$$\underbrace{\frac{1}{\lambda}}_{\text{first pass used}} + \underbrace{\frac{1}{\lambda}\left(\underbrace{\frac{\lambda-1}{\lambda}}_{\text{first pass non-used}}\right)}_{\text{Second pass using the "first pass non-used"}} \qquad (1)$$

This would give $\dfrac{2\lambda-1}{\lambda^2}$ (2)

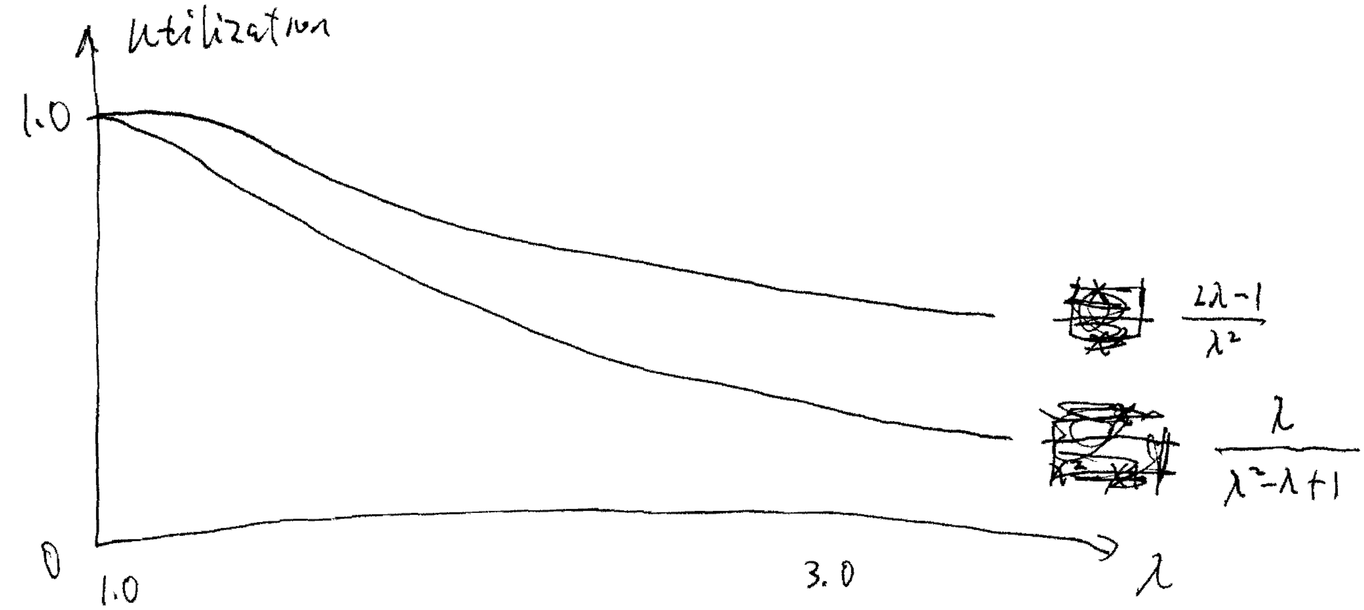
This results didn't consider the dilution effect of the recycle, and is therefore not 100% right in the sense of asymptotic behavior when $\lambda \to \infty$

② The addition in (1) needs to be weighted with the concentration difference.

In general. we can find the overall expression by dimensional analysis and asymptotic behavior.

ⓐ Note. λ is from $1 \sim \infty$. so in general. the expression needs to be linear in λ in the numerator and quadratic in the denominator, actully. (2) fits in this demand nicely.

ⓑ check $\frac{\lambda}{\lambda^2-\lambda+1}$, when $\lambda \to \infty$ the expression $\to \frac{1}{\lambda}$, fit our definition, when $\lambda \to 1$, $\frac{\lambda}{\lambda^2-\lambda+1} \to 1$



Therefore, $\frac{\lambda}{\lambda^2-\lambda+1}$ is a better results, because at large λ, it gives $\frac{1}{\lambda}$. while the ~~latte~~ $\frac{2\lambda-1}{\lambda^2}$ gives $\frac{2}{\lambda}$

$\frac{2\lambda-1}{\lambda^2}$ is more applicable to the case the fuel is cascaded to two fuel cells. while $\frac{\lambda}{\lambda^2-\lambda+1}$ is better withe fuel circulating back into the inlets again. which should not matter withe recirculation when the λ is big (utilization is small).

Open book Q4

Note Slope $= \frac{RT}{\alpha n F}$ but the intercept of the line on the $\ln|j|$ axis is $\ln j_0$ only!

the number $\frac{RT}{\alpha n F}\ln(j_0)$ is the intercept on the $\eta(V)$ axis which really depends on where one takes the origin of the $\ln|j|$.

So the intercept is $\ln(j_0)$ but not $\frac{RT}{\alpha n F}\ln(j_0)$

The T dependance of $\frac{RT}{\alpha n F}$ is clear there.

As for $\ln j_0$, $j_0 = nFc_R^* f_1 e^{-\Delta G_1^\ddagger/RT}$

$$\frac{d\ln j_0}{dT} = \frac{+\Delta G_1^\ddagger}{R}\cdot\frac{+1}{T^2}$$

The pressure dependence of the both comes from the Nerst equation, in ~~that~~ activation barrier depends on the concentration and pressure.

the change in $\Delta G_i^{\ddagger}$ is give by $\frac{RT}{nF} \ln\left(\frac{P}{P_0}\right)^{\Delta n_g}$

Therefore $\frac{d \ln j_0}{dp} = \frac{d \frac{-\Delta G_i^{\ddagger}}{RT}}{dp} = \frac{d \frac{1}{nF} \ln\left(\frac{P}{P_0}\right)^{\Delta n_g}}{dp}$

Very similarly, the slope is given by

$$b = \frac{RT}{\alpha nF} - \frac{RT}{nF}\left[\ln\left(\frac{P}{P_{std}}\right)^{\Delta n_g}\right]$$

differentiate the above vs. P should give the P dependence.

The End.

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